Robustness Checks in Structural Analysis

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Outline

I am going to do three things!

- Explain what is going on
- Identify an issue and offer a solution
- Talk about the importance of the topic

Find an easy way to do robustness checks regarding moment selection in structural estimation.

- Why does this need to be easier?
- Structural estimation is sssslllloooowwww.
- It is hard to do robustness if it takes one week to do an estimation.

Why is structural estimation slow

- Estimate some parameters, $\boldsymbol{\theta}$, of an **economic** model by minimizing

$$\left[\begin{array}{r}\text{data}\\ \text{moments}\end{array} - \begin{array}{r}\text{simulated}\\ \text{moments}\end{array}\left(\boldsymbol{\theta}\right)\right]' \hat{\boldsymbol{W}} \left[\begin{array}{r}\text{data}\\ \text{moments}\end{array} - \begin{array}{r}\text{simulated}\\ \text{moments}\end{array}\left(\boldsymbol{\theta}\right)\right]$$

weight matrix

- simulated moments $(\boldsymbol{\theta})$ can be very nonlinear
- no closed form

- Minimizing

$$\left[\begin{array}{r}\text{data}\\ \text{moments}\end{array} - \begin{array}{r}\text{simulated}\\ \text{moments}\end{array}\left(\boldsymbol{\theta}\right)\right]' \hat{\boldsymbol{W}} \left[\begin{array}{r}\text{data}\\ \text{moments}\end{array} - \begin{array}{r}\text{simulated}\\ \text{moments}\end{array}\left(\boldsymbol{\theta}\right)\right]$$

is like trying to walk down:

- blindfolded

Walking down the mountain takes time

- No closed form
- With each step you have to
  - solve the model
  - simulate data
  - recalculate the bumpy thing to be minimized
- It can take 50,000 or more steps.
- Estimating once is usually fine. Many times? Not so much.

Illustrate their alternative with a very simple investment model

- A firm maximizes the expected present value of shareholder distributions
- Distributions are profits minus investment
- Profits come from an exogenous shock and a decreasing returns function of capital
- The firm invests in capital

$$\max_{k_{t+j}, j=0,\dots,\infty} \quad \mathbb{E}_0 \sum_{j=0}^{\infty} \frac{1}{(1+r)^t} \left(z_{t+j} k_{t+j}^{\alpha} - \left(k_{t+j+1} - (1-\delta) k_{t+j} \right) \right)$$

$$\max_{k_{t+j}, j=0,\dots,\infty} \quad \mathbb{E}_0 \sum_{j=0}^{\infty} \frac{1}{(1+r)^t} \left(\text{profits} \quad - \text{investment} \right)$$

The solution is "marginal product of capital equals user cost!"

- It is that simple

$$
\begin{aligned}
E_t\left(\alpha z_{t+1} k_{t+1}^{\alpha-1}\right) &= r+\delta \\
E_t\left(\mathrm{MPK}_{t+1}\right) &= \text{interest rate} + \text{depreciation rate} \\
&= \text{user cost}
\end{aligned}
$$

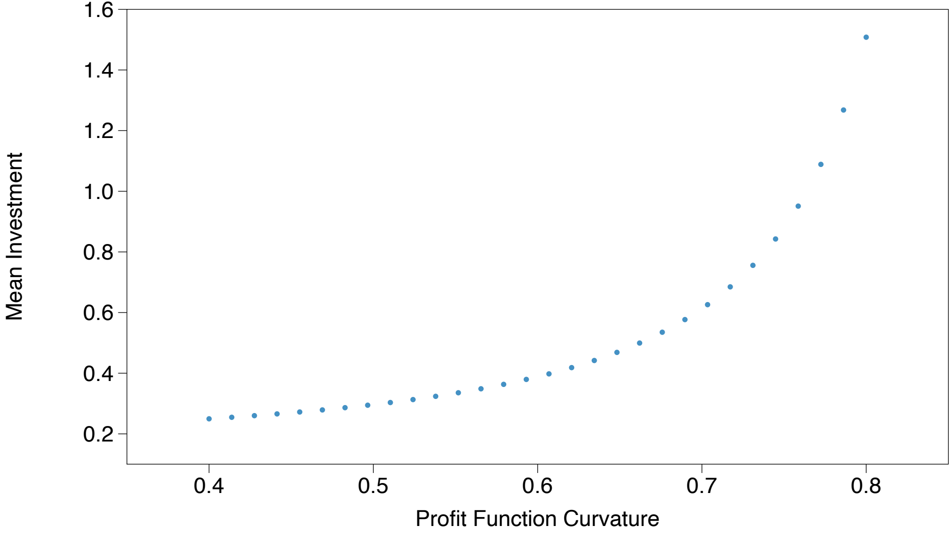
I want to estimate profit function curvature (α)

- Let's say I know δ, r, and the driving process for z_t.
- And I estimate α
- But ... I am unsure if I picked the right moments.

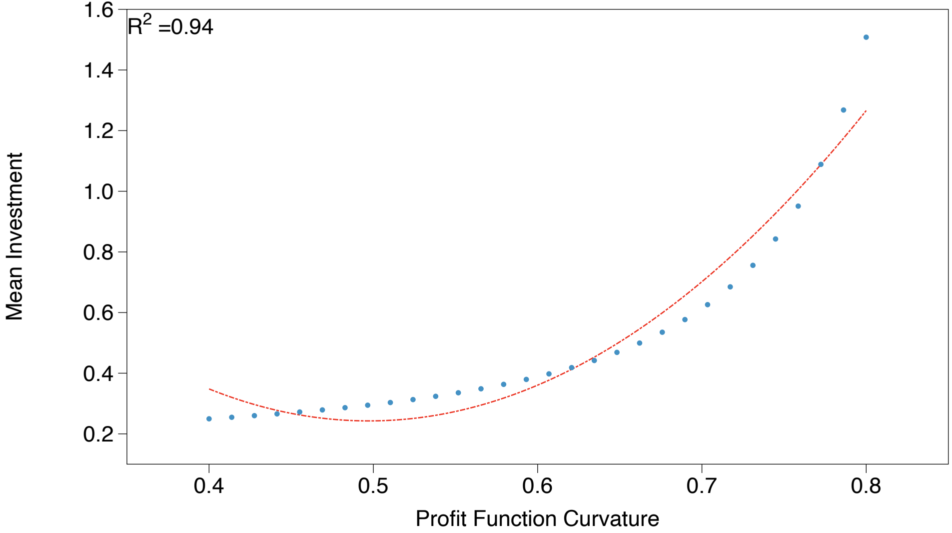
I could just reestimate using all possible combinations of available moments

- Or I could use their method
  - Start by choosing carefully selected, random-ish values for α (Sobol' or Halton sequences)
  - For each I solve and simulate the model and calculate moments.

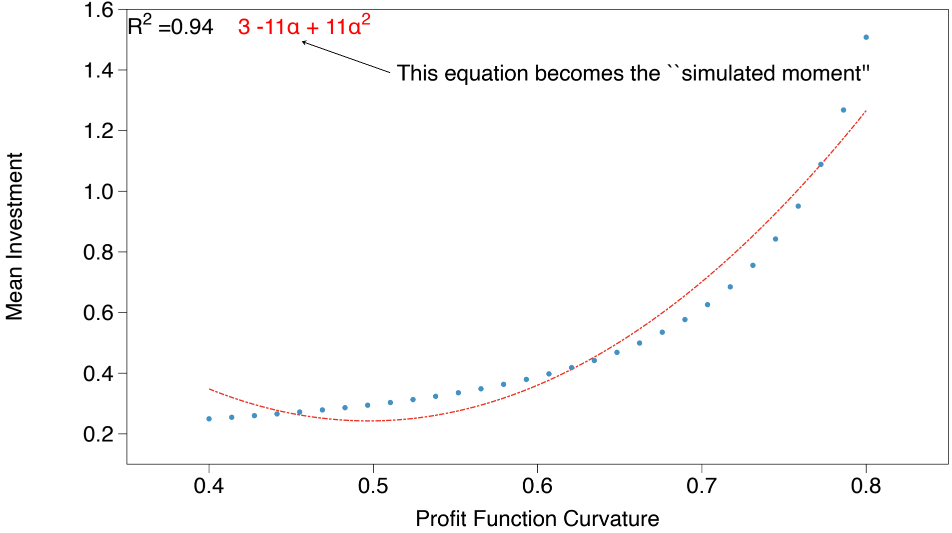
So I make the moments for each value of α



And I fit the moment to the parameter



And I fit the moment to the parameter



Do the same thing for a bunch of other moments

- Before, computing the objective function was a process:
- Now!

$$\left[\begin{array}{r}\text{data}\\ \text{moments}\end{array} - \begin{array}{r}\text{approximate}\\ \text{moments}\end{array}\left(\boldsymbol{\theta}\right)\right]' \hat{\boldsymbol{W}} \left[\begin{array}{r}\text{data}\\ \text{moments}\end{array} - \begin{array}{r}\text{approximate}\\ \text{moments}\end{array}\left(\boldsymbol{\theta}\right)\right]$$

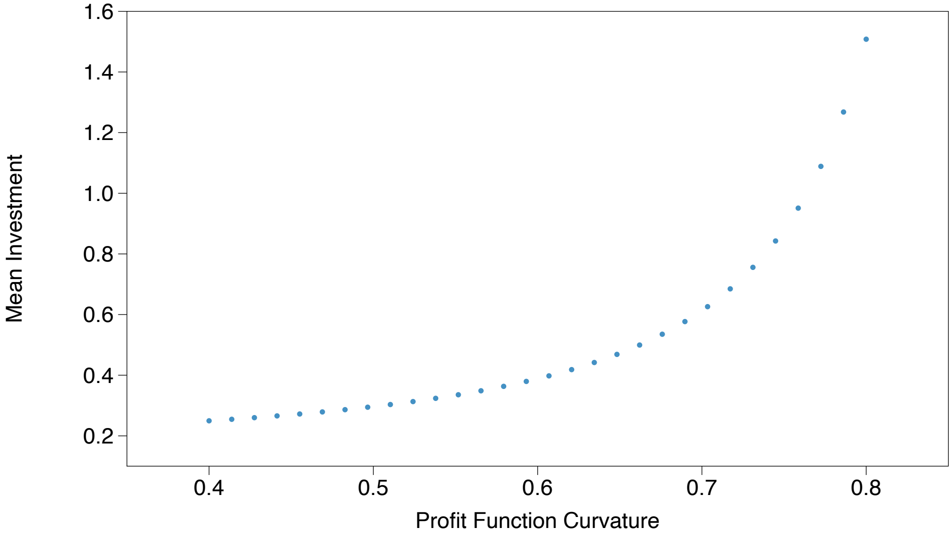
- has a closed form
- easy to try all possible moment vectors

Outline

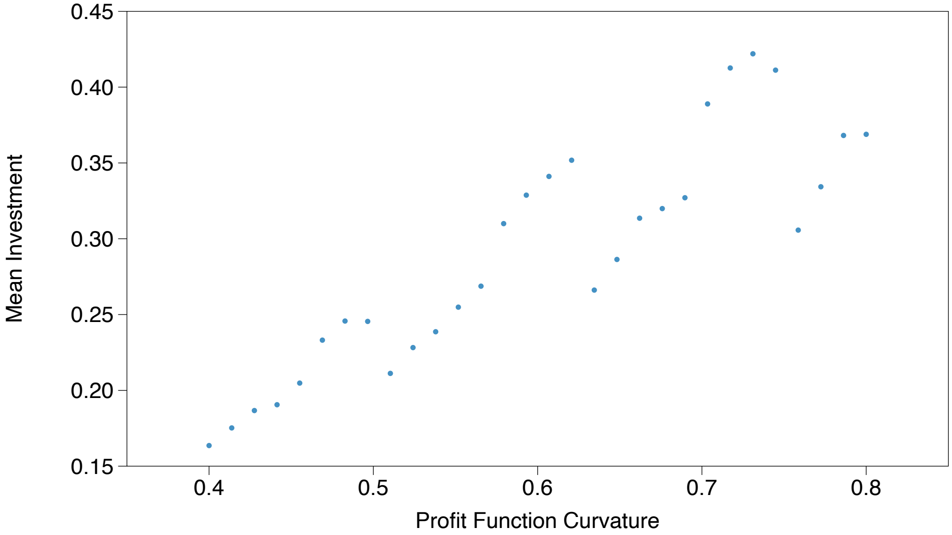
Especially in corporate finance, these plots are often bumpy

- Frictions and nonconvexities are the bread and butter of corporate finance
- I am going to add some nonconvexities to the model
  - Non-negative profit constraint (no equity issuance)
  - Fixed cost of adjusting the capital stock

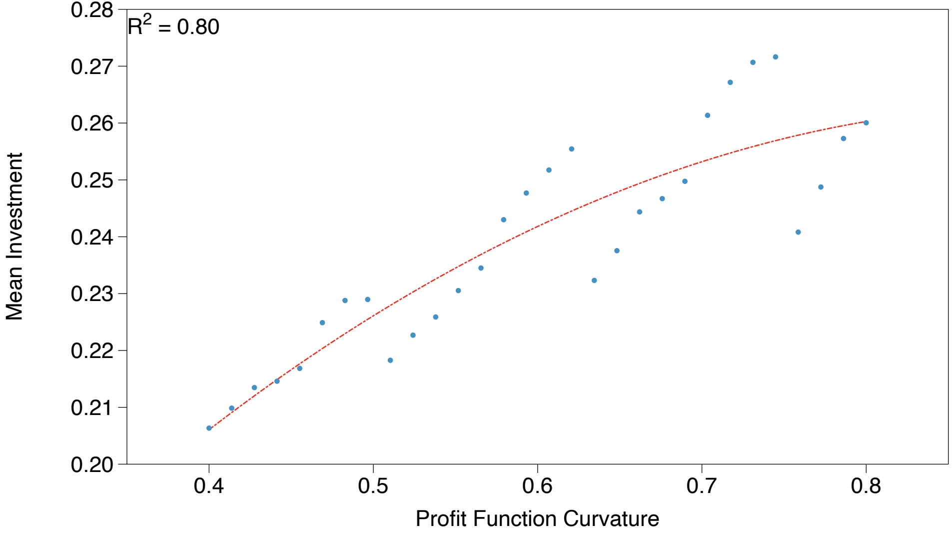
Remember how the plot used to be smooth?



It's not anymore with the nonconvexities



And the fit is not as good



But is this serious? I answer with Monte Carlos

- How often does approximate SMM recover the true value?
- Two Monte Carlos:
  - Using the actual bumpy, non-closed-form moment functions
  - Using the smooth, closed-form approximate moment functions
- Just with one specific choice of moments
  - Mean investment
  - Variance of investment

Monte Carlo Design

- Solve model at $\alpha = 0.7$.
- Simulate 1000 small data sets ($n = 500$)
- Estimate the curvature (α) parameter using actual and approximate SMM:

The results are not great for approximate SMM

- Approximate SMM is screamingly fast.
- And very accurate at hitting the wrong spot.
- All results are expressed as a fraction of the true value

	Actual SMM	Approx SMM Quadratic	Approx SMM Cubic
Mean Bias	0.009	0.279	0.163
RMSE	0.040	0.294	0.163

The authors should do a bunch of Monte Carlos

- Look at models with **serious nonconvexities**
- See how good an approximation you need to **make the estimator work.**
- You can use a **laptop.**

The authors do look at the accuracy of the approximations

- But not directly at the implications for estimation.
- This would improve the paper a great deal.

Outline

Is moment choice the right dimension of robustness?

- Referees never ask me to check robustness with respect to moment choice.
- Models should **not** be "robust" to the choice of moments.
- They are models, not reality.
- They **will** break.

The set of moments should be tailored to the economic question

- If your question is about **A** . . .
- and you can match moments related to **A** . . .
- and you are transparent about the model's inevitable limitations,
- that's as good as it can get

A more important dimension of robustness is the **model**

- All referees ask me about this question.
- Most structural robustness sections are about this question.
- It's the right question to ask.

Outline

The method in this paper might end up being quite useful

- I want to know how much **estimation** error it introduces
- I want to know if you can reduce this error with better approximations
- I do not think the technique will be useful for "robustness."
- The technique might be useful to identify where models break.
- We can **learn** something from where the models break down.